

The $a_0(980)$, $a_0(1450)$ and $K_0^*(1430)$ Scalar Decay Constants and the Isovector Scalar Spectrum

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Abstract

The scalar correlator $\Pi(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T \left(J(x) J^\dagger(0) \right) | 0 \rangle$ (with $J = \partial_\mu (\bar{d} \gamma^\mu u)$) is studied using a class of finite energy sum rules shown recently to be very well satisfied in the vector isovector channel. The values of the a_0 scalar decay constants extracted in this analysis, which describe the couplings of the $a_0(980)$ and $a_0(1450)$ to the current J , are shown to be comparable, strongly disfavoring any scenario in which the $a_0(980)$ is interpreted as a loosely bound $K\bar{K}$ molecule and the $a_0(1450)$ is assigned to the same flavor multiplet as the $K_0^*(1430)$. The a_0 decay constants are also compared to the analogous decay constant describing the coupling of the $K_0^*(1430)$ to the divergence of the strange vector current, $J_s = \partial_\mu (\bar{s} \gamma^\mu u)$ (which may be obtained from experimental K_{e3} and $K\pi$ phase shift data) and implications of the relative magnitudes for the interpretation of the nature of the a_0 states discussed.

14.40.Cs, 12.39.Mk, 11.55.Hx, 11.40.-q

I. INTRODUCTION

Despite considerable recent theoretical activity, there exists, at present, no consensus on the nature of the $f_0(980)$ and $a_0(980)$ mesons. Interpretations still advocated in the literature include (1) the $q^2\bar{q}^2$ (“four-quark”) cryptoexotic interpretation originally proposed by Jaffe [1] (see Refs. [2,3]); (2) the $K\bar{K}$ bound state, or “molecule”, picture [4–8]; (3) the unitarized quark model picture [9–13,8]; and (4) the “minion” picture of Gribov [14].

The $K\bar{K}$ molecule assignment for the $f_0(980)$ and $a_0(980)$ is attractive (and, perhaps for this reason, often cited as already established) because it naturally explains the proximity to $K\bar{K}$ threshold and strong $K\bar{K}$ couplings of the states. Such bound states occur naturally in the quark model treatment of Ref. [4], as well as several coupled channel treatments using model meson-meson interactions with parameters fit to experimental data [5–8]. The picture predicts $\gamma\gamma$ decay widths [17,18,7] compatible with experiment [15,16], and considerably smaller than those expected, at least in the simplest version of the quark model, for 3P_0 $q\bar{q}$ mesons. One should bear in mind, however, that an alternate quark model approach, whose $\gamma\gamma$ model dynamics are constrained by an analogous treatment of $\pi^0, \eta \rightarrow \gamma\gamma$, allows widths compatible with experiment for a conventional quark model assignment [19], and that small $\gamma\gamma$ widths are also claimed in the four-quark picture, based on a schematic estimate loosely motivated by the MIT bag model [20]. An alternate means of testing the molecule scenario is via the processes $\phi \rightarrow f_0\gamma, a_0\gamma$ [21]. Recent experimental determinations [22,23] give branching ratios larger than predicted in the molecule picture [21,24] but, since the predictions of Refs. [21] and [24] differ by a factor of 4, realistic theoretical uncertainties may be large, making the discrepancy with experiment difficult to interpret reliably.

One of the problems with the interpretation of the light scalar states is that their narrow experimental widths, which appear unnatural for a conventional $q\bar{q}$ meson assignment (but natural in the $K\bar{K}$ molecule picture), may not, in fact, reflect the true intrinsic widths, since strong coupling of an intrinsically broad state to a nearby s -wave threshold can produce significant narrowing through the effects of channel coupling and unitarity [25]. Given the empirically observed strong $f_0(980)$ and $a_0(980)$ couplings to $K\bar{K}$, the possibility arises that these states are, not molecules, but rather “unitarized quark model” (UQM) states, i.e. conventional $q\bar{q}$ mesons with properties strongly distorted by coupling to the nearby $K\bar{K}$ threshold [9]. In such a scenario, one expects significant $K\bar{K}$ content in the scalar states, and hence most likely a significant reduction in the $\gamma\gamma$ widths [9] (though these widths have yet to be calculated in the UQM picture). The distortion caused by the strong coupling will presumably also alter expectations for $\phi \rightarrow f_0\gamma, a_0\gamma$, though one again awaits an explicit calculation of this effect.

A number of observations made in the context of the UQM, and discussions surrounding it, bear repeating at this point. First, in the presence of strong coupling to nearby decay channels, a single underlying state can manifest itself as more than just a single pole close to the physical region (Törnqvist and Roos [9], for example, claim that both the $a_0(980)$ and $a_0(1450)$ are likely to be manifestations of the same underlying $q\bar{q}$ state). Second, existing experimental data are apparently insufficient to distinguish between the molecule and UQM pictures, since coupled channel models exist, both with and without an underlying conventional quark model state, which provide good fits to existing data in the coupled $\pi\pi, K\bar{K}$ and $\pi\eta, K\bar{K}$ channels, once the data has been used to fit the free parameters of the

models. Third, although the method for distinguishing conventional resonances from bound states posited by Morgan and Pennington [11] (a conventional resonance having poles near the physical region on both the second and third sheets, a bound state having such a pole on only the second sheet) is frequently borne out in existing coupled channel calculations [4–8], this is not universally the case [8]. In particular, a situation in which there are two nearby poles, one on the second and one on the third sheet, can arise in a model which provides a good fit to existing data, but for which one of the nearby poles is most naturally thought of as the coupled-channel remnant of a $K\bar{K}$ bound state (in the sense that, as the channel coupling is dialed up towards its final fitted value, what was a $K\bar{K}$ bound state in the absence of channel coupling moves continuously to become one of the two final nearby poles) [8]. Thus, even if data could distinguish between the one-nearby-pole and two-nearby-pole scenarios (see Refs. [11,5,26] for arguments that this may not be possible at present), it appears that it would not allow us to distinguish between the UQM and molecule scenarios.

In this paper, we discuss the feasibility of using meson decay constants describing the coupling of the $I = 1/2$ and $I = 1$ scalar mesons to (pointlike) flavor-non-diagonal scalar densities, as a means of further clarifying the nature of the isovector scalar spectrum. The basic idea is the same as that underlying attempts to make use of the processes $f_0, a_0 \rightarrow \gamma\gamma$ and $\phi \rightarrow f_0\gamma, a_0\gamma$, namely that the spatial extent of a loosely bound $K\bar{K}$ molecule is significantly larger than that of a conventional $q\bar{q}$ meson, which is, in turn, significantly larger than that of the very compact Gribov minions [14]. The scalar densities, being pointlike, provide a direct probe of such scale differences, and one which is, moreover, less prone to obscuration by intervening dynamics. In other channels, analogous decay constants provide useful information on the classification of states. For example, the fact that, using recent ALEPH data, one has [27]

$$g_{K^{*-}} = 1.1 g_{\rho^-} \simeq g_{\rho^-} , \quad (1)$$

(where $g_{V^{ab}}$, with $a, b = u, d, s$, is defined by $\langle 0 | J_\mu^{ab} | V^{ab} \rangle \equiv g_{V^{ab}} \epsilon_\mu$, with $J_\mu^{ab} = \bar{q}_a \gamma_\mu q_b$, V^{ab} the corresponding vector meson, and ϵ_μ the vector meson polarization vector) supports the assignment of the ρ and K^* to the same $SU(3)_F$ multiplet. The same-multiplet assignment of the π and K is similarly supported by the result

$$f_K = 1.2 f_\pi \simeq f_\pi , \quad (2)$$

while the fact that

$$g_\omega^{EM} / g_{\rho^0}^{EM} \simeq 1/3 , \quad (3)$$

(where $g_\omega^{EM}, g_{\rho^0}^{EM}$ are the electromagnetic decay constants of the ω and ρ^0), rather than the value $\simeq 1/\sqrt{3}$ (expected if the ω is the flavor 8 member of a vector meson octet), gives direct evidence for ideal mixing in the vector meson nonet.

Expectations for the relative sizes of the scalar decay constants in the various scenarios are obvious: they should be small for a weakly bound $K\bar{K}$ molecule, large for the very compact ($\sim .1 - .2$ fm) Gribov minions and, for conventional $q\bar{q}$ states, should obey approximate $SU(3)_F$ relations among the the decay constants of different members of the same multiplet. In the four-quark picture, since, in the bag model, a four-quark state is larger

than a two-quark state and, in addition, the hidden strange pair would have to be annihilated, one would also expect the decay constant to be suppressed. We will show below that experimental data allows us to determine the scalar decay constant of the $K_0^*(1430)$, while a QCD sum rule analysis fixes the analogous $a_0(980)$ and $a_0(1450)$ decay constants. We will then show that (1) the weakly bound $K\bar{K}$ molecule and minion scenarios are ruled out by the sum rule analysis and (2) that the relations between the various decay constants do not support the assignment of either a_0 resonance as the $SU(3)_F$ partner of the $K_0^*(1430)$, but rather suggest a UQM-like scenario.

II. THE $K_0^*(1430)$ SCALAR DECAY CONSTANT

We define the decay constants of interest to us in this paper as follows:

$$\langle 0 | \partial^\mu J_\mu^{su} | K_0^*(1430) \rangle \equiv f_{K_0^*(1430)} m_{K_0^*(1430)}^2 \quad (4)$$

$$\langle 0 | \partial^\mu J_\mu^{du} | a_0 \rangle \equiv f'_{a_0} m_{a_0}^2 \quad (5)$$

where, in Eq. (5), a_0 refers to either of the two a_0 resonances. In QCD, one has

$$\partial^\mu J_\mu^{ab} = i(m_a - m_b) S^{ab} , \quad (6)$$

with $S^{ab} = \bar{q}_a q_b$. Since the scalar densities, S^{ab} , are members of an $SU(3)_F$ octet, $SU(3)_F$ relations are simplified by redefining the a_0 decay constants in such a way that a common mass factor occurs on the LHS's of Eqs. (4) and (5), i.e.,

$$\left(\frac{m_s - m_u}{m_d - m_u} \right) \langle 0 | \partial^\mu J_\mu^{du} | a_0 \rangle \equiv f_{a_0} m_{a_0}^2 . \quad (7)$$

For an a_0 state lying in the same $SU(3)_F$ multiplet as the $K_0^*(1430)$, one should then have

$$f_{a_0} m_{a_0}^2 \simeq f_{K_0^*(1430)} m_{K_0^*(1430)}^2 , \quad (8)$$

while, in the $K\bar{K}$ molecule/four-quark/minion pictures, the LHS of Eq. (8) should be, respectively, much smaller/smaller/larger than the RHS.

To make use of the expectation provided by Eq. (8), one first requires the decay constant of the “reference” quark model state, the $K_0^*(1430)$. This may be obtained from the spectral function, $\rho_s(s)$, of the correlator, $\Pi_s(q^2)$, defined by (with $J_s = \partial^\mu J_\mu^{su}$),

$$\Pi_s(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T \left(J_s(x) J_s^\dagger(0) \right) | 0 \rangle , \quad (9)$$

since one has, at the $K_0^*(1430)$ peak, neglecting background,

$$\frac{f_{K_0^*(1430)}^2 m_{K_0^*(1430)}^3}{\pi \Gamma_{K_0^*(1430)}} = \rho_s(m_{K_0^*(1430)}^2) . \quad (10)$$

$\rho_s(s)$ may, in turn, be constructed from experimental $K\pi$ phase shifts and the value of the timelike scalar $K\pi$ form factor, $d(s)$, at some convenient kinematic point [28,29] since (1) unitarity relates the $K\pi$ component of the spectral function to $d(s)$ via

$$[\rho_s(s)]_{(K\pi)} = \frac{3}{32\pi^2} \sqrt{\frac{(s-s_+)(s-s_-)}{s^2}} |d(s)|, \quad (11)$$

(where $s_{\pm} = (m_K \pm m_{\pi})^2$) and, (2) $d(s)$ satisfies an Omnes relation,

$$d(s) = d(0) \exp \left[\frac{s}{\pi} \int_{th}^{\infty} ds' \frac{\delta(s')}{s'(s' - s - i\epsilon)} \right], \quad (12)$$

where $\delta(s)$ is the phase of $d(s)$, and the normalization $d(0)$ is known from ChPT and K_{e3} [31]. (In writing this equation, we have, as elsewhere in the literature, ignored a possible polynomial pre-factor; see Ref. [30] for an empirical justification.) Experimentally, $K\pi$ scattering is known to be purely elastic up to $s \sim 2.5 \text{ GeV}^2$ [32–34]. Thus, $\delta(s)$ is identical to the $I = 1/2$ $K\pi$ phase shift up to this point. At the edge of the experimental region, $s = (1.7 \text{ GeV})^2$, moreover, the measured $K\pi$ phase has essentially reached the asymptotic value, π , required by quark counting rules for $d(s)$, allowing one to rather safely assume $\delta(s) = \pi$ for $s > (1.7 \text{ GeV})^2$. With these assumptions, $[\rho_s(s)]_{K\pi}$ is determined by Eqs. (11) and (12). The self-consistency of these assumptions has been checked by the sum rule analysis of Ref. [30]. Finally, the $K_0^*(1430)$ $K\pi$ branching fraction is known to be compatible with 100% [34], so the $K\pi$ component represents the full spectral function in the $K_0^*(1430)$ region. (Note that Particle Data Group values for the mass and width [15] reflect mis-transcriptions in Ref. [33]; the corrected values are $m_{K_0^*(1430)} = 1.412 \text{ GeV}$ and $\Gamma_{K_0^*(1430)} = 0.294 \text{ GeV}$ [34]. The corresponding values of the effective range parameters appearing in the LASS parametrization of the $K\pi$ phase are $a = 2.19 \text{ GeV}^{-1}$ and $b = 3.74 \text{ GeV}^{-1}$ [34].) Combining experimental data and Eqs. (10), (11) and (12), one obtains

$$f_{K_0^*(1430)} m_{K_0^*(1430)}^2 = .0842 \pm .0045 \text{ GeV}^3. \quad (13)$$

The errors in Eq. (13) reflect the range of values obtained when the $K_0^*(1430)$ resonance parameters are varied within the errors quoted for the phase shift fit of Ref. [28], and also the difference between the values obtained using the corrected LASS fit and the fit of Ref. [28].

III. THE ISOVECTOR SCALAR DECAY CONSTANTS

The spectral function, $\rho(s)$, of the isovector scalar correlator, $\Pi(q^2)$, receives contributions from the two a_0 resonances proportional to $f_{a_0(980)}^2$ and $f_{a_0(1450)}^2$, respectively. This makes a QCD sum rule treatment of $\Pi(q^2)$, in which the OPE of $\Pi(q^2)$ is used to fix unknown parameters of the hadronic spectral function, such as $f_{a_0}^2$, an especially favorable way to distinguish between “large decay constant” and “small decay constant” scenarios.

Previous attempts to determine $f_{a_0(980)}$ using QCD sum rules employed the conventional Borel transformed (SVZ) sum rule method [35] (see Section 7.3 of Ref. [36] for details). Borel transformation of the original dispersion relation converts the weight in the hadronic spectral integral into an exponentially falling one, $\exp(-s/M^2)$, where the Borel mass, M , is a parameter of the transformation, and, simultaneously, both kills subtraction constants and provides factorial suppression of higher dimension condensate contributions on the OPE side ($1/(Q^2)^n \rightarrow 1/(n-1)!M^{2n}$, with $Q^2 = -q^2 = -s$). Small values of M suppress dependence of the transformed spectral integral on the unknown high- s portion of the spectral function,

while large values suppress dependence of the OPE side on unknown higher dimension condensates. Ideally, one finds a “stability plateau (or window)”, i.e., a region of M values for which both suppressions are reasonably operable, and within which extracted spectral parameters are roughly constant. Since, typically, the high- s contribution to the transformed spectral integral is not negligible in the stability window, one requires a model for this portion of the spectral function. The standard approach is to use the “continuum ansatz”, or local duality approximation, for $\rho(s)$, for all s beyond some “continuum threshold”, s_0 . This is known to be a crude approximation in regions where typical resonance widths are not much greater than typical resonance separations, and hence leads to systematic uncertainties if continuum contributions to the spectral integral are significant. In the case of the earlier sum rule analyses of $f_{a_0(980)}$, one sees, from Fig. 7.6 of Ref. [36], both the absence of a stability plateau and a strong sensitivity to the choice of s_0 . The latter observation signals the importance of contributions from the continuum region.

An alternate implementation of the QCD sum rule approach, which avoids the use of the local duality approximation, is the finite energy sum rule (FESR) method. Consider the so-called “PAC-man” contour, which traverses both sides of the physical cut between threshold, s_{th} , and s_0 , on the timelike $q^2 = s$ axis, and is closed by the circle of radius s_0 in the complex s -plane. Cauchy’s theorem and analyticity then ensure that

$$\int_{s_{th}}^{s_0} ds \rho(s) w(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds w(s) \Pi(s) , \quad (14)$$

for any function $w(s)$ analytic in the region of the contour. Typically one wishes to use spectral data and/or a spectral ansatz on the LHS of Eq. (14), and the OPE on the RHS. The isovector scalar channel is very well adapted to the FESR approach, as far as the hadronic side of the sum rule is concerned, since the first two resonances in the channel are rather well-separated. Thus, if one chooses s_0 so as to include only the first two resonance regions, a well-motivated, resonance-dominated spectral ansatz is possible. Since the resonance masses and widths are known, only the decay constants remain as unknown parameters. The potential problem with the FESR approach is that those s_0 for which one can reliably employ a resonance-dominated ansatz for $\rho(s)$ correspond, by definition, to scales for which local duality is not a good approximation. This means, in particular, that the OPE representation for $\Pi(s)$ on the circle $|s| = s_0$ must, at the very least, break down over some region near the timelike real axis. Fortunately, an argument by Poggio, Quinn and Weinberg, suggests that, for moderate s_0 , this is the only region over which this breakdown should occur [37]. This is confirmed empirically by the success of the conventional FESR treatment of hadronic τ decays [38] (which involves a weight, determined by kinematics, having a double zero at $s = s_0 = m_\tau^2$), and by the investigation of Ref. [39], which shows that FESR’s involving weights $w(s)$ having either a single or double zero at $s = s_0$ are all extremely well satisfied in the isovector vector channel (where one can use the experimentally determined spectral function [27,40]), even at scales, s_0 , significantly below m_τ^2 . In Figure 1 we show that such “pinch-weighted” FESR’s can be used to very accurately extract hadronic spectral parameters. In the Figure, the dots (with error bars) are the experimental ALEPH data [27]; the dashed line shows the result of using the OPE side of a continuous family of such sum rules to fix the decay constants appearing in a simple spectral ansatz consisting of an incoherent sum of three Breit-Wigner resonances [39]; and the solid line represents a least squares fit

of the same spectral ansatz directly to the data. Note the very accurate determination of the $\rho(770)$ decay constant. This determination is dominated by the perturbative ($D = 0$) contributions to the OPE, and hence by the value of the running coupling, α_s . While a determination of a non-perturbative quantity like g_ρ in terms of a perturbative one like α_s might sound implausible, we remind the reader that this is possible because we have used empirical non-perturbative information (in the form of resonance masses and widths) as input to the sum rules. Once this information has been input, analyticity relates the running coupling and the resonance decay constants via the basic FESR relation Eq. (14).

In light of above discussion, we chose to study the isovector scalar channel using the pinch-weighted FESR method. For the spectral function we employ a sum of $a_0(980)$ and $a_0(1450)$ contributions, using experimental input for the masses and widths. s_0 is then restricted to lie at or below the upper edge of the second resonance region (actually, somewhat higher, since the zero at $s = s_0$ means that the region just below s_0 in the spectral integral has negligible weight). We take the maximum value of s_0 to be 3.0 GeV^2 .

On the OPE side, the expressions for the dimension $D = 0, 2, 4$ and 6 contributions are given in Refs. [28,41], and will not be reproduced here. The $D = 0$ (perturbative) contribution is known to four-loop order, and is determined by the running light quark masses and coupling. The light quark mass scale is set by any one of m_u, m_d, m_s , since the mass ratios are determined by ChPT [42]. We employ, as our basic input, the value $m_s(2 \text{ GeV}) = 115 \pm 8 \text{ MeV}$ (quoted in the \overline{MS} scheme) determined in Ref. [30]. Ratios of decay constants are, of course, independent of this choice. As input for the running coupling we take $\alpha_s(m_\tau^2) = 0.334 \pm 0.022$ [27,40]. The masses and couplings at other scales are obtained from the exact solutions of the RG equations generated using the four-loop truncated versions of the β [43] and γ [44] functions. As input for the higher dimension condensate contributions, we employ conventional values: $\langle \alpha_s G^2 \rangle = 0.07 \pm .01 \text{ GeV}^4$ [45], $(m_u + m_d) \langle \bar{u}u \rangle = -f_\pi^2 m_\pi^2$, and $\langle g \bar{q} \sigma F q \rangle = (0.8 \pm 0.2 \text{ GeV}^2) \langle \bar{q}q \rangle$ [28]. The four-quark $D = 6$ condensate terms are taken to have their vacuum saturation values, modified by an overall multiplicative factor, ρ_{VSA} . To be conservative, we consider the range $\rho_{VSA} = 5 \pm 5$, allowing, therefore, up to an order of magnitude violation of vacuum saturation. Integrals around the circle $|s| = s_0$ on the OPE side of Eq. (14) are all performed using the contour improvement prescription of LeDiberber and Pich [46], which is known to improve convergence, and reduce residual scale dependence of the truncated perturbative series. Since the convergence of the integrated $D = 0$ contributions worsens as s_0 is lowered, we have chosen $s_0 = 2.4 \text{ GeV}^2$ as a minimum value for our analysis. Ideally, to improve convergence, one would like to know the mass and width of the third a_0 resonance, and then work at larger scales s_0 , but this is not possible at present. We discuss our estimate of the resulting truncation errors below.

The last point in need of discussion is the treatment of instanton effects. These effects can be important in scalar and pseudoscalar channels, especially at lower scales such as those we have been forced to work at here. The effect of instantons on $\Pi(s)$ is known exactly only in the approximation in which one treats the single instanton configuration in the background of the perturbative vacuum [47,48]. It is known, however, that quark and gluon condensates strongly affect the density for large scale instantons [49]. Since contributions in Refs. [47,48], from instantons large enough to be subject to this effect are known to be significant [47], the exact results are of only schematic use (as pointed out by the authors themselves). An alternate representation of instanton effects is provided by the instanton liquid model,

in which the effective instanton density is assumed to be sharply peaked around a single effective average size [50]. Although this sharp peaking leads to a slower fall-off with q^2 than would be obtained using a broader distribution of sizes [51], the model has the advantage of being phenomenologically constrained [50]. We have, therefore, employed the instanton liquid model. Ref. [52] gives the form of the corresponding contributions to FESR's with $w(s) = s^k$. In view of the the phenomenological nature of the model, we chose, to be safe, to restrict ourselves to weights $w(s)$ for which instanton effects are not large. In order that this suppression not be specific to the particular q^2 -dependence of the instanton liquid model, we further restrict ourselves to those weights for which an evaluation using the much more strongly q^2 -dependent form obtained in Ref. [48] are also small. This turns out to restrict us to weights of the form

$$w(s) = \left(1 - \frac{s}{s_0}\right) \left(A - \frac{s}{s_0}\right), \quad (15)$$

with the parameter A lying in the range $\simeq 2 \pm 1$. The suppression of the incompletely known instanton contributions is optimal for $A = 2$, and we display all results below employing this value. (Note that the weight $w(s)$ has been constructed so as to have a zero at $s = s_0$, which is required in order that the resulting FESR's be reliable at scales such as that considered here [39].) We will use the difference between the results obtained using the two different instanton implementations as a (hopefully conservative) measure of the uncertainty associated with our use of the instanton liquid model.

Fitting the $a_0(980)$ and $a_0(1450)$ decay constants by matching the hadronic and OPE sides of the FESR above in the fit window $2.4 \text{ GeV}^2 < s_0 < 3.0 \text{ GeV}^2$, we then obtain, adding errors from all sources in quadrature,

$$f_{a_0(980)} m_{a_0(980)}^2 = 0.0447 \pm 0.0085 \text{ GeV}^3 \quad (16)$$

$$f_{a_0(1450)} m_{a_0(1450)}^2 = 0.0647 \pm 0.0123 \text{ GeV}^3. \quad (17)$$

Uncertainties due to those on the resonance masses and widths are completely negligible; the errors, therefore, reflect uncertainties in the input to the OPE side of the sum rules. The major sources of error on $f_{a_0(980)} m_{a_0(980)}^2$ are as follows: (1) from that on $\alpha_s(m_\tau^2)$: $\pm 0.0068 \text{ GeV}^3$, (2) from that on the overall mass scale, $(m_s - m_u)^2$: $\pm 0.0032 \text{ GeV}^3$, (3) from that on ρ_{VSA} : $\pm 0.0023 \text{ GeV}^3$ and (4) due to truncation of the perturbative series: $\pm 0.0029 \text{ GeV}^3$. Those on $f_{a_0(1450)} m_{a_0(1450)}^2$ are, similarly: (1) from that on $\alpha_s(m_\tau^2)$: $\pm 0.0112 \text{ GeV}^3$, (2) from that on the overall mass scale: $\pm 0.0046 \text{ GeV}^3$, (3) from that on ρ_{VSA} : $\pm 0.0015 \text{ GeV}^3$ and (4) due to truncation of the perturbative series: $\pm 0.0011 \text{ GeV}^3$. If one employs, instead of the instanton liquid model, the results of Ref. [48], $f_{a_0(980)} m_{a_0(980)}^2$ is increased by 0.0044 GeV^3 and $f_{a_0(1450)} m_{a_0(1450)}^2$ by 0.0023 GeV^3 . We remind the reader that the instanton liquid model is subject to phenomenological constraints, while the results of Ref. [48] are not.

One final source of uncertainty, not included in the errors quoted above, is relevant in the case of the $a_0(1450)$. Because the $a_0(980)$ is well separated from subsequent resonances in the channel, its extracted decay constant is stable with respect to assumptions about the behavior of the spectral function beyond 3 GeV^2 . This is, however, not necessarily true for the $a_0(1450)$, which might, for example, overlap to some extent, and hence interfere with, the next higher resonance. Even were this interference to be incoherent, there could still

be a non-trivial contribution from the tail of the next a_0 under the $a_0(1450)$. This would lead to an overestimate of the $a_0(1450)$ decay constant. We cannot meaningfully investigate this uncertainty since we do not know the width, or location, of the next a_0 resonance. In order to get a feel for the resulting uncertainty, however, we have investigated the effect of including a third resonance in the spectral ansatz, taking, for illustration, its mass and width to be 1.9 GeV and 0.4 GeV, respectively. Assuming the contributions add incoherently, one finds that the $a_0(1450)$ decay constant is decreased by $\sim 40\%$. That level of uncertainty in the $a_0(1450)$ decay constant is, therefore, unavoidable without further experimental input. In contrast, the $a_0(980)$ decay constant is, as expected, essentially unaffected by the use of the three-resonance ansatz.

IV. DISCUSSION

For results obtained using QCD sum rules to be considered reliable, it is necessary that the form of the spectral ansatz employed be physically sensible. Even the most ridiculous spectral ansatz will have some choice of parameters which “optimizes” the match between the OPE and hadronic sides. Of course, if the ansatz is not a good one, this “optimal” match will be poor. While at the scales we have employed, resonance dominance seems a very safe assumption, it is worthwhile checking this statement. This is done in Figure 2. The Figure displays the results for the $A = 2$ pinch-weighted FESR discussed above, both for the two-resonance ansatz on which the results above are based, and for two other ansätze, which serve as the basis for further discussions below.

In the Figure, the dashed-dotted line represents the OPE side of the sum rule, the dotted line the optimized match of the hadronic to the OPE side, obtained by adjusting the two a_0 decay constants. The agreement is obviously excellent. The solid line, in contrast, represents the match obtained when one forces $f_{a_0(980)}^2 \simeq 0$ by hand (as would be expected for a loosely-bound $K\bar{K}$ molecule) and adjusts $f_{a_0(1450)}^2$ to optimize the match. The resulting fit is very poor, ruling out the loosely-bound $K\bar{K}$ interpretation of the $a_0(980)$. Similarly, the fact that no good match exists with $f_{a_0(980)}$ much larger than $f_{K_0^*(1430)}$ rules out the minion interpretation. A sceptical reader might object that the poor fit between the OPE and hadronic sides in the $a_0(1450)$ -only spectral ansatz might be cured, not only by the addition of a narrow a_0 contribution, but also by the inclusion of a broader non-resonant background, even though such a possibility appears somewhat perverse, in view of the extremely good match between the dashed-dotted and dotted curves. The dashed line in the Figure demonstrates that this possibility is also ruled out, for reasons we will now explain.

Although we do not know, in detail, what to expect for the background $\pi\eta$ and $K\bar{K}$ contributions to the isovector scalar spectral function below the $a_0(1450)$ region, it is rather easy to make sensible rough estimates. Indeed, once one knows the scalar form factors describing the couplings of the scalar densities to the $\pi\eta$ and $K\bar{K}$ states, one obtains the corresponding contributions to the spectral function by unitarity, as in Eq. (11). Near threshold, the timelike form factors may be trivially computed using ChPT. If one performs this exercise for the $\pi\eta$ contribution, using the tree-level ChPT expression for the timelike form-factor, the optimized background-plus- $a_0(1450)$ fit is very close to that of the solid line in Figure 2. The dashed line is the result of multiplying this background contribution by a

factor of 5, and then adjusting $f_{a_0(1450)}$ so as to optimize the match between the OPE and hadronic sides (but still with no explicit $a_0(980)$ contribution). Again one sees that the fit is very poor, demonstrating that it is a narrow state, and not a broad background which is required in the spectrum at low s .

We, thus, conclude that the results obtained above are, indeed, reliable, subject to the quoted errors and the caveats regarding the $a_0(1450)$ decay constant already discussed. We see that (1) the products $f_{a_0} m_{a_0}^2$ for the two a_0 resonances are comparable and (2) that both are somewhat smaller than the corresponding product for the $K_0^*(1430)$. In addition to ruling out the $K\bar{K}$ molecule and minion pictures, these results suggest a UQM-like scenario. One ambiguity which is unavoidable concerns the way in which one interprets the expected $SU(3)_F$ relations amongst the decay constants for the states lying in the same multiplet as the $K_0^*(1430)$. In the $SU(3)_F$ limit, of course, all states in the same multiplet would have the same mass, so whether one compared the values of $f m^2$ or the values of f would make no difference. In attempting to decide whether or not the $a_0(980)$ should be interpreted as the $I = 1$ partner of the $K_0^*(1430)$, however, this ambiguity plays a potentially significant role: if we compare decay constants, then we have

$$\frac{f_{a_0(980)}}{f_{K_0^*(1430)}} = 1.1 \quad (18)$$

whereas, if we compare the products of the decay constants and the squared masses, we have

$$\frac{f_{a_0(980)} m_{a_0(980)}^2}{f_{K_0^*(1430)} m_{K_0^*(1430)}^2} = 0.53 . \quad (19)$$

Since, in other channels, it is the strange state which has the larger decay constant, however, neither of these results, in fact, corresponds to what one might expect based on the pattern from other meson nonets.

In conclusion, we have determined the scalar decay constants of the $a_0(980)$, $a_0(1450)$ and $K_0^*(1430)$ mesons. The relations between them suggest a UQM-like scenario for the isovector scalar states. At present, we do cannot be certain that the results rule out the four-quark interpretation of the $a_0(980)$, though it appears likely that one would expect a much smaller value for $f_{a_0(980)}$ in this scenario. A calculation of the $f_{a_0(980)}$ in the four-quark picture would thus be highly desirable.

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FIGURES

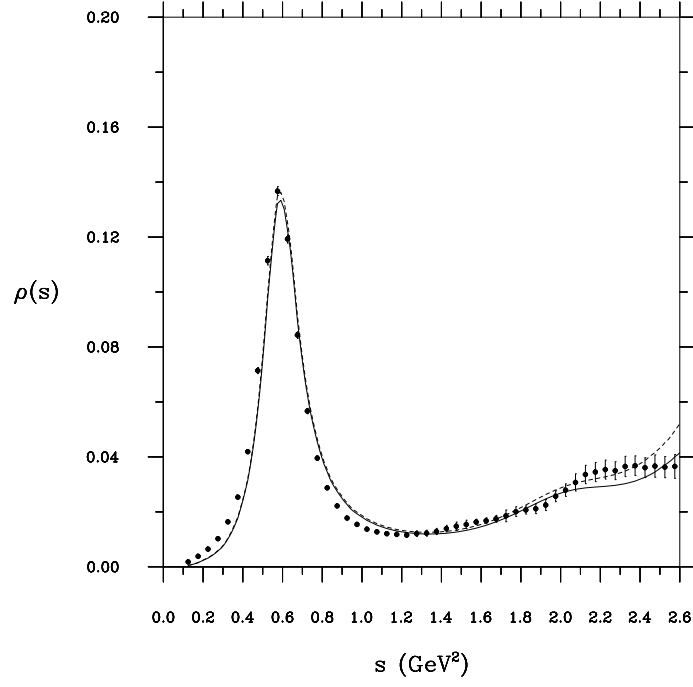


FIG. 1. Comparison of data, the representation generated by fitting to the OPE, and the least squares fit to data, for the isovector vector spectral function. See the text for details.

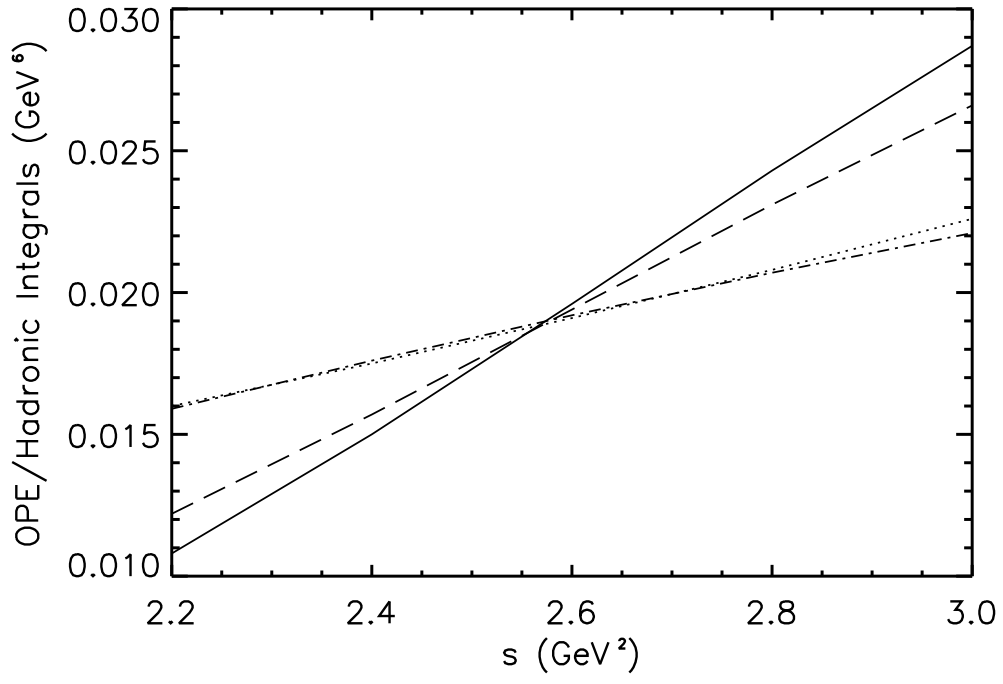


FIG. 2. The $A = 2$ pinch-weighted FESR. The curves correspond to the OPE side of the sum rule and the hadronic sides corresponding to the three different spectral ansätze described in the text.

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